

**Dynamic System Models  
for Flood Hydrograph  
Prediction and Control**

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FOR FLOOD HYDROGRAPH  
PREDICTION AND CONTROL

Ministry  
of the  
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Water Resources  
Paper 9

Dynamic System Models for  
Flood Hydrograph Prediction and Control

by

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Toronto, September 1977

DYNAMIC SYSTEM MODELS FOR  
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L. A. Logan

ABSTRACT

Dynamic system models of linear and nonlinear hydrologic responses, as related to excess rainfall runoff processes, are employed and compared for flood hydrograph estimation, prediction and control. The linear response model is represented by the Dimensionless Instantaneous Unit Hydrograph and the nonlinear response model is based on a second order differential equation which includes the effects upon storage of unsteady flows and excess rainfall. Parameter estimations for these deterministic models are achieved with the application of direct search optimization techniques. A minimum variance Kalman filter algorithm is employed for the sequential estimation of the output and parameter of the linear model, in the presence of noise disturbances. In addition, a deterministic iterative estimator algorithm is used with the linear model in a recursive prediction of flows by way of minimizing the residual with respect to a dimensionless parameter as the control variable. The nonlinear model is examined on the basis of response surfaces for paired parameters and on the basis of the model output sensitivity to varying the parameters. Subsequently response noise models are determined and examined for possible typical real time model application. A satisfactory application of the nonlinear model was achieved in the prediction of the downstream hydrograph with input from excess rainfall and regulated dam releases.

# DYNAMIC SYSTEM MODELS FOR FLOOD HYDROGRAPH PREDICTIONS AND CONTROL

by  
L.A. Logan<sup>1</sup>

## INTRODUCTION

Many of the hydrologic models that are currently available for flood flow predictions are mainly linear and deterministic with time invariant parameters. These models are often inadequate for some flow prediction and planning purposes because they do not, as a rule, include any explicit consideration of the uncertainties that are inherent in the form of a model and estimates of its parameters. In the real world, the rainfall-runoff process is truly nonlinear, nonstationary and is imbedded with both process and measurement noise disturbances. Although the probability characteristics of these noise processes may not be known or are known only partially, for practical purposes, they may be assumed to be zero mean Gaussian white noise processes (Jazwinski, (1970)).

Early models, based on the Instantaneous Unit Hydrograph concept (IUH) (Nash (1957), Dooge (1959)) are linear and deterministic. A nonlinear IUH problem was treated by Singh (1964), and a reproduction of the excess rainfall-runoff process on the basis of a nonlinear reservoir storage model with time invariant parameters was achieved by Prasad (1967). Amoroch (1967), examined the general problem of nonlinear model predictions in the study of the rainfall-runoff process with special reference to the problems of predictive uncertainties.

Recently, Hino (1973, 1975), Lee (1972), employed state estimation techniques of the modern Control Theory in the sequential estimation of parameters and states for linear and nonlinear hydrologic system response models. These approaches were attempts to account for some aspects of the stochastic nature of the response processes by including noise disturbances in solving the estimation problem. Duong, et al, (1975), and Szollosi-Nagy (1976), extended the Control Theory application in hydrology to include adaptive estimation and control of the input noise levels.

This paper utilizes linear and nonlinear reservoir storage models in comparison for flood hydrograph predictions. Recursive filter algorithms are employed in the sequential estimation, with noise dynamics, of parameters and flows by the linear model. In addition, iterative estimation and prediction of flows are attempted by way of minimizing the mean squared residual with respect to the dimensionless model parameter as the control variable. The nonlinear model is tested for its applicability in the prediction of hydrographs with controlled input in the presence of excess rainfall. The objective is to implement simple but more realistic and effective models for flood hydrograph predictions and control.

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### Response Models

The storage discharge relation for surface runoff processes, in the presence of disturbance effects on storage by unsteady flow and excess rainfall, is combined with the hydrologic continuity equation to produce a second order, nonlinear, differential equation representing a lumped nonlinear hydrologic response model:

$$[\alpha_2 D^2 + N \alpha_1 q^{N-1} D + 1] q = [1 - \alpha_3 D] u \dots [1]$$

where  $q$  and  $u$  denote runoff output and excess rainfall input respectively;  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  are storage coefficients for flow, unsteady flow and rainfall excess, respectively;  $N$  is a constant accounting for the nonlinearity of the system relative to lumped storage and discharge; and  $D \equiv d/dt$ , is a differential operator, with  $dq/dt$ , defining the unsteady flow and  $du/dt$ , defining the rainfall intensity.

By assuming linearity of the system ( $N=1$ ) and negligible disturbance effect on storage, that is, uniform flow,  $dq/dt = 0$ , and uniform excess rainfall intensity,  $du/dt = 0$ , eq. [1] reduces to a linear, first order differential equation for the response model:

$$[\alpha_1 D + 1] q_t = u_t \dots [2]$$

The solution of eq. [2] with unit step input, for equal  $n$  linear reservoirs in series and with the same time invariant parameter, results in the familiar Instantaneous Unit Hydrograph (IUH) (Nash 1957). The IUH may be reduced to the dimensionless IUH (Wu, (1963)):

$$q/q_p = [t/t_p]^{n-1} e^{-(n-1)(t/t_p - 1)} \dots [3]$$

where  $q_p$  and  $t_p$  denote peak flow and time-to-peak, respectively. Equation [3] represents the unique functional for the linear model considered.

A finite difference scheme may be employed in the numerical solution of eq. [1]. The resulting algorithm with rearrangement to solve for  $q(j+1)$  is:

$$q(j+1) = \left[ \frac{\alpha_2}{\Delta t^2} + \frac{\alpha_1 N}{2\Delta t} q^{N-1}(j) \right]^{-1} \cdot \left[ u(j+1) - \frac{\alpha_3}{\Delta t} (u(j+1) - u(j)) \right. \\ \left. - q(j) \left( 1 - \frac{2\alpha_2}{\Delta t^2} \right) - q(j-1) \left( \frac{\alpha_2}{\Delta t^2} - \frac{\alpha_1 N}{2\Delta t} q^{N-1}(j) \right) \right] \dots [4]$$

in which a central difference scheme is used for  $q$ , and a forward difference scheme for  $u$ .

### Dynamic System Models

The equivalent dynamic system models may be represented by the following process and observation models (Jazwinski (1970)):

$$\frac{dX(t)}{dt} = f(X, U; t) + G(t) W(t), \quad X(t_0) = X_0 \dots [5]$$

$$Z(t) = h(X, U; t) + V(t) \dots [6]$$

where  $X$  and  $U$  denote  $n$ -vector state output and  $p$ -vector input respectively;

$f(X, U; t)$  and  $G(t)$  denote  $n$ -vector functional characterizing the system and a noise coefficient matrix, respectively;  $W(t)$  is a  $p$ -vector process noise;

$Z(t)$  is a  $m$ -vector observed output;  $h(X, U; t)$  is a nonlinear functional characterizing the output and  $V(t)$  denotes a  $r$ -vector measurement noise.

$W(t)$  and  $V(t)$  are assumed to be uncorrelated zero mean Gaussian white noise processes, with covariance matrices,  $Q$  and  $R$ , respectively.

A recursive Kalman filter algorithm may be employed as an alternative technique to the conventional trial-and-error or least square methods in solving the estimation problem in real time application. For example, a discrete, minimum variance filter for the linear model, eq. [3], based on eqs. [5] and [6], comprises a conditional mean estimate (Jazwinski (1970), Sage and Melsa (1971)):

$$\hat{X}(k|k) = \hat{X}(k|k-1) + K[Z(k) - H\hat{X}(k|k-1)] \dots [7]$$

in which for the augmented system:

$$\hat{X}(k|k) \equiv \begin{bmatrix} \hat{q}(k|k) \\ \hat{n}(k|k) \end{bmatrix} \dots [8]$$

$H^T = \begin{bmatrix} 1 & 0 \end{bmatrix}$ , denotes the observation matrix; and  $K$ , denotes the steady state optimal Kalman gain matrix given by the Riccati equations:

$$K = A P H^T [H P H^T + R]^{-1} \dots [9]$$

and the steady state error covariance  $P$ , defined by:

$$P = A P A^T - K H P A^T + \Gamma Q \Gamma^T \dots [10]$$

where  $A$  and  $\Gamma$  denote the state coefficient matrix and noise coefficient matrix, respectively, for a discrete, linear, time invariant system.

The sequential estimator results in  $A(k)$ ,  $H(k)$ ,  $\Gamma(k)$  and  $K(k)$  as updated matrices at each observation. Similarly, the covariance matrices  $Q(k)$  and  $R(k)$ , if required can be updated at each observation (Jazwinski (1970), Duong, et al (1975)).

### Noise Control Models

In practice, the forms of the response models are usually inadequate and only estimates of the true values of model parameters are known; hence the residuals are often not truly white noise uncorrelated. A suitable noise model may be fitted to the residuals and used to modify or augment the response model in prediction. Improvement in predictions is achieved when the residuals of the augmented response noise model are approximately white noise uncorrelated.

A suitable noise model of the residual may be incorporated into the model algorithm to provide optimal feedback control adjustments to the input  $u_t$  as the control variable. The effect of the unobserved disturbance,  $N_t$ , may be cancelled by setting (Box and Jenkins (1976)):

$$\Delta u_t = -v^{-1}(B)(1-\phi_1 B)N_{t+} \dots [11]$$

where  $\Delta u_t$  denotes the adjustment to be made on  $u_t$  at time  $t$ ; and  $N_{t+}$ , denotes the model residual of the one-step-ahead prediction;  $V(B)$  is the impulse response function weights and  $\phi_1 B$  denotes the lag-1 autoregressive noise model parameter.

A suitable response noise model and feedback control algorithm may be developed for the linear or nonlinear model.

### APPLICATION AND DISCUSSION

The response models are evaluated with experimental data in the Wilmot Creek IHD Representative drainage basin. Test applications were subsequently done in other selected project areas. Optimal estimates of the parameters for the deterministic forms of the models were determined with the use of direct search optimization techniques. "Golden ratio" technique was employed with the linear model and a "pattern search" technique was used with the nonlinear model. The minimum variance Kalman filter algorithm was also used in solving the linear estimation problem, with the consideration of associated process and measurement noise disturbances. Control noise model of the residuals was examined in relation to the nonlinear model.

#### Linear Models

As a result of the test applications in the Wilmot Creek basin, there appear to be sufficient variations in the model parameters, particularly parameter  $n$ , to indicate that these variations are due, probably, to parameter uncertainties as a result of influences by the process and measurement noise disturbances associated with the estimation problem.

The recursive prediction with iterative estimation of the dimensionless parameter  $n$  as a control variable is demonstrated in Figure 1.1. The autocorrelation plots of the residuals indicated that the successive values of the residuals are approximately uncorrelated, in comparison with those of the predictions with the optimal value for  $n$ . The results of this control algorithms are satisfactory and although the method may be suitable for real time application, it is however, still dependent on the accuracy of the estimates of  $q_p$  and  $t_p$  to be determined by alternative empirical equations in relation to the accuracy of rainfall forecasts.

Figures 1.2, 1.3 and 1.4 show the results of the application of the linear minimum variance filter for an alternative, one-step-ahead prediction with the consideration of input noise disturbances. The results of the sequential estimation of flows based on constant input noise levels are satisfactory. However, there is an appreciable improvement in predictions with an adaptive estimation of the input noise levels. The latter approach gave the more realistic results, in attempting to compensate for model error by way of adapting the noise levels to the residuals. In addition, Figures 1.3 and 1.4 demonstrate the stochastic nature of the sequential estimation as indicated by the propagation of the values for the steady state gains,  $K$ , and error covariances,  $P$ , respectively. In this example of the adaptive estimation,  $Q$  and  $R$  tend to converge very quickly to stable values from the initial guesses.

The sequential minimum variance filter approach is to be favoured over the above deterministic recursive approach, seeing that in addition to providing simultaneous estimates of the state and parameter, it determines, also, estimates of the error covariances, from which reliability and accuracy of the predictions can be established.

#### Nonlinear Model

Preliminary analysis with the nonlinear model indicates that explicit consideration of nonuniform rainfall intensity, in solving the estimation problem, appears to be significant only in the larger catchments with a large  $\alpha_2$  storage coefficient and in application with controlled inputs. Figure 2.1 shows examples of response surfaces for paired parameters. The parameters  $\alpha_1$  vs  $\alpha_2$  and  $N$  vs  $\alpha_2$  display similar response surfaces with a distinct local minimum. However, parameters  $N$  vs  $\alpha_1$ , appear to have both a critical and sensitive relationship with respect to the objective function and describe a surface which appears to comprise contours approaching that of an elliptical hollow.

The sensitivity of the model output to the varying of each of the model parameters, in turn, is demonstrated in Figure 2.2. The parameter  $\alpha_2$  influences and is directly proportional to the lag time to the peak flow. Both parameters  $\alpha_1$  and  $N$  influence the magnitude of the peak. For given optimal constants  $\alpha_2$  and  $N$ , smaller values for  $\alpha_1$  are related to larger peaks and vice versa. However, for this example, in which the optimal  $\alpha_1 < 1$  and in relation to a small value for  $N$ , it is observed that with this combination for given constant  $\alpha_2$ , a decrease in the value of  $N$  is associated with larger peaks and vice versa. However, in other applications, in which optimized parameter  $\alpha_1 > 1$ , a larger value of  $N$  is related to larger peak flows and vice versa. Figure 2.3(A) shows an example of the model application in a larger catchment, resulting in larger storage coefficients  $\alpha_1$  and  $\alpha_2$ .

Figure 2.3(B) shows the result of the nonlinear model application in demonstrating its applicability in a typical problem solution. Here the upstream dam excess releases are regarded as controlled inputs and are used in addition to the excess rainfall and tributary inputs to generate the related downstream hydrograph. In this example the effect of storage coefficient,  $\alpha_3$ , is included in the model calibration and prediction. Sensitivity analysis has indicated that  $\alpha_3$  has direct influences on both the magnitude of the peak flow and time-to-peak (Figure 2.4(A)).

Further applications, with the inclusion of the control noise model algorithm, results in significant improvement in the reproduction of the hydrograph, particularly in cases with less than satisfactory deterministic calibration (Figure 2.4(B)).

Table 1 summarizes the parameters and flow statistics for the nonlinear model application with a number of rainfall-runoff events.

#### CONCLUSIONS

The optimal values determined for the parameter,  $n$ , of the linear model, from its application to several storm events, exhibit significant variations even within a given catchment. Similarly, the hydrograph predictions with a constant optimal,  $n$ , gave less than satisfactory results. However, significant improvement in prediction is achieved with the use of a recursive and iterative estimation algorithm in which the model parameter,  $n$ , is treated as a control variable. Alternatively, the sequential estimation of the parameter,  $n$ , and prediction of flows, with the use of the minimum variance Kalman filter algorithm, attempted to account for imbedded noise disturbances in the solution of the problem.

Further improvements in prediction are achieved as a result of attempts to compensate for some model error by the use of the residuals in an adaptive estimation of the noise covariances as part of the solution.

The nonlinear response model applications resulted in satisfactory hydrograph predictions. However, the optimized parameters are, similarly, unique to a given storm event. The variations in the value of each parameter within a given catchment are probably due to the inherent system and observation noises associated with the modelling problem. There appears to be a consistent response surface relationship between paired optimized parameters, with the exception of paired parameters  $\alpha_1$  vs  $N$ . The optimization of the model is very sensitive to the proper combination of these parameters. The consideration of the effect of nonuniform input upon storage in the estimation problem appeared to be more significant in combination with controlled inputs. Satisfactory estimation of model parameters and hydrograph prediction are achieved with the combination of excess inputs from control dam discharges, tributary flows and excess rainfall.

Examples of suitable response noise models are established for some selected catchment and storm situations. These could serve as preliminary models in simulation experiments for developing suitable control algorithms in attempts to compensate for model errors in the estimation problem.

The sequential estimation and prediction algorithms are ideal for on-line implementation in solving real time flood forecasting problems. The combined nonlinear response and input control noise model, once suitably implemented for a particular problem site, could serve as a useful screening model in monitoring control of dam or reservoir releases in relation to any projected or target downstream rainfall-runoff hydrograph.

# REFERENCES

- Amorocho, J. (1967): The Nonlinear Prediction Problem in the Study of the Runoff Cycle, Water Resources Research, Vol. 3, No. 3 pp. 861-880.
- Box, G. and Jenkins, G. (1976): Time Series Analysis-Forecasting and Control, Holden-Day pp. 423.
- Dooge, J. C. (1959): A General Theory of Unit Hydrograph, Journal of Geophysical Research, Vol. 64, No. 1, pp. 241-256.
- Duong, N. Winn, C. and Johnson, G., (1975): Modern Control Concepts in Hydrology, IEEE Transaction on Systems, Man, and Cybernetics, Vol. SMC-5 No. 1, pp. 46-53.
- Hino, M. (1973): On-Line Prediction of Hydrologic System, Proceedings of the 15th Congress of the International Association for Hydraulic Research, Vol. 4, pp. 121-129.
- Hino, M. (1975): Estimation of Discharge Characteristics in Rapidly Changing Hydrologic Basin, Symposium on the Hydrological Characteristics of River Basins, Tokyo, IAHS Publication No. 117, pp. 313-319.
- Jazwinski, A. (1970): Stochastic Process and Filtering Theory, Academic Press, New York.
- Lee, S. (1972): Analysis, Modelling and Forecasting of Water Quality Systems - Nonlinear Filtering and Estimation, Vol. II, The Kansas Water Resources Research Institute, Contribution No. 110.
- Nash, J. (1957): The Form of Instantaneous Unit Hydrograph, International Association of Scientific Hydrology, Vol. 3, pp. 241-256.
- Prasad, R. (1967): A Nonlinear Hydrologic System Response Model, Journal of the Hydrologic Division, Proceedings ASCE, No. Hy4, pp. 201-221.
- Sage and Melsa, (1971): Estimation Theory with Application to Communication and Control, McGraw-Hill Book Co.
- Singh, K. (1964): Nonlinear Instantaneous Unit Hydrograph Theory, Journal of the Hydraulic Division, Proceeding ASCE, Vol. 90 No. Hy2, pp. 313-347.
- Szollosi-Nagy, A. (1976): An Adaptive Identification and Prediction Algorithm for Real-Time Forecasting of Hydrological Time Series, Hydrological Sciences - Bulletin XX1, No. 13, pp. 16-176.
- Wu, I-Pai. (1963): Design of Hydrograph for Small Watersheds in Indiana, Journal of the Hydraulic Division, Proceedings ASCE, No. Hy6, pp. 35-66.

Table 1: Summary Statistics of the Nonlinear Model Applications.

| Basin<br>(Area)                           | Storm<br>Date | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | N    | $\phi_1$ | $S_e$<br>(mm) | $q_p$ (mm/h)<br>meas. cal. | $t_p$ (h)<br>meas. cal. | Depth (mm)<br>meas. cal. |
|-------------------------------------------|---------------|------------|------------|------------|------|----------|---------------|----------------------------|-------------------------|--------------------------|
| Nithburg<br>(326.3 km <sup>2</sup> )      | 23/8/75       | 10.2       | 54.1       | ~          | 0.7  | -        | 0.13          | 2.3 2.1                    | 16 16                   | 40.0 39.2                |
|                                           | #23/8/75      | 10.2       | 59.1       | -0.53      | 0.7  | -0.03    | 0.12          | 2.3 2.1                    | 16 16                   | 40.0 39.4                |
| New Hamburg<br>(551.7 km <sup>2</sup> )   | 23/8/75       | 13.8       | 152.9      | ~          | 0.05 | -        | 0.15          | 1.4 1.2                    | 22 23                   | 27.7 29.0                |
|                                           | 23/8/75       | 7.8        | 145.6      | 0.83       | 0.10 | -        | 0.14          | 1.4 1.2                    | 22 23                   | 27.7 29.0                |
| St. Jacobs<br>(212.4 km <sup>2</sup> )*   | 16/5/74       | 6.6        | 7.3        | ~          | 0.56 | -        | 0.64          | 9.0 6.6                    | 19 20                   | 91.9 86.1                |
|                                           | 16/5/74       | 6.8        | 8.0        | -0.23      | 0.55 | -        | 0.63          | 9.0 6.6                    | 19 20                   | 91.9 85.6                |
|                                           | #16/5/74      | 6.8        | 8.0        | -0.23      | 0.55 | 0.83     | 0.39          | 9.0 7.8                    | 19 19                   | 91.9 93.8                |
| West Montrose<br>(194.3 km <sup>2</sup> ) | 16/5/74       | 5.7        | 0.41       | 1.0        | 0.48 | -        | 0.34          | 11.7 11.0                  | 18 19                   | 146.3 144.7              |
|                                           | #16/5/74      | 5.7        | 0.41       | 1.0        | 0.48 | 0.30     | 0.31          | 11.7 11.1                  | 18 18                   | 146.3 144.7              |

$\alpha_1$  - storage coefficient - flow  
 $\alpha_2$  - " " - unsteady flow  
 $\alpha_3$  - " " - input  
N - nonlinear parameter  
 $\phi_1$  - noise-model parameter  
 $S_e$  - standard error of estimate

$q_p$  - peak flow  
 $t_p$  - time-to-peak flow  
vol - volume of surface runoff  
~ - assumed zero for  $\alpha_3$   
\* - drainage area: partial, up to the dam location  
# - with control noise-model

|                                                                                                                          |   |                                              |
|--------------------------------------------------------------------------------------------------------------------------|---|----------------------------------------------|
| Nith/Nithburg (02GA038)<br>Nith/New Hamburg (02GA018)<br>Conestoga/St. Jacobs (02GA006)<br>Grand/West Montrose (02GA034) | } | Southern Ontario Streamflow gauging stations |
|--------------------------------------------------------------------------------------------------------------------------|---|----------------------------------------------|

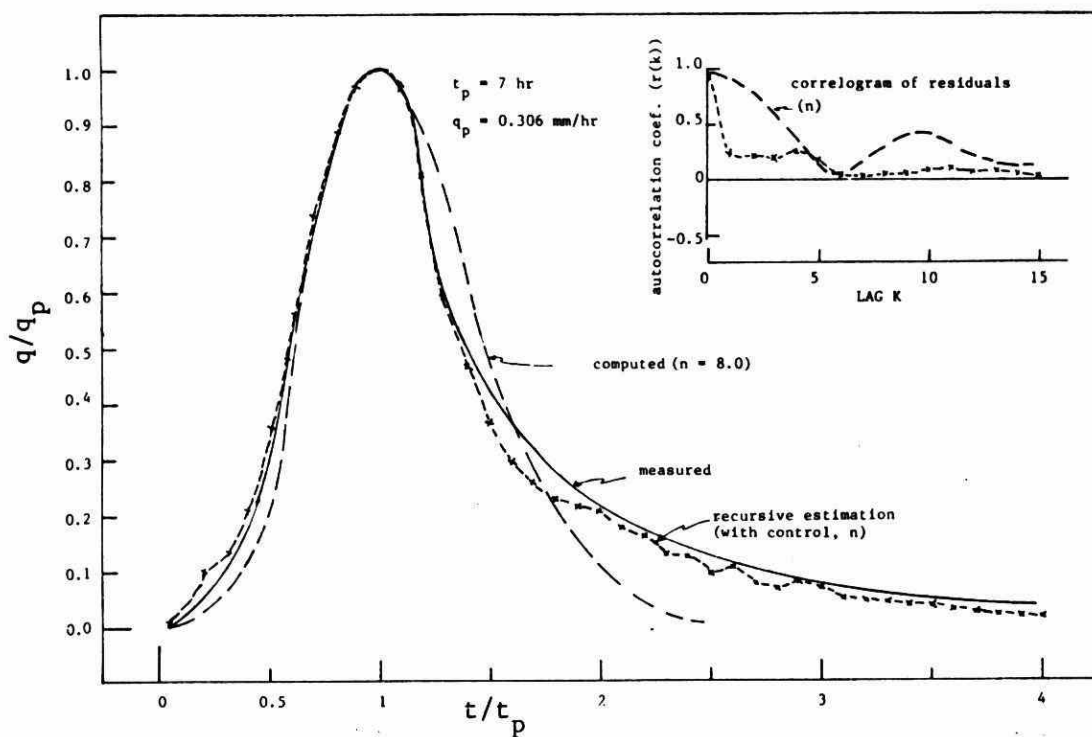


Figure 1.1: Hydrograph Prediction: Linear Model (DIUH)  
(Wilmot Creek Basin - Event 16/8/69).

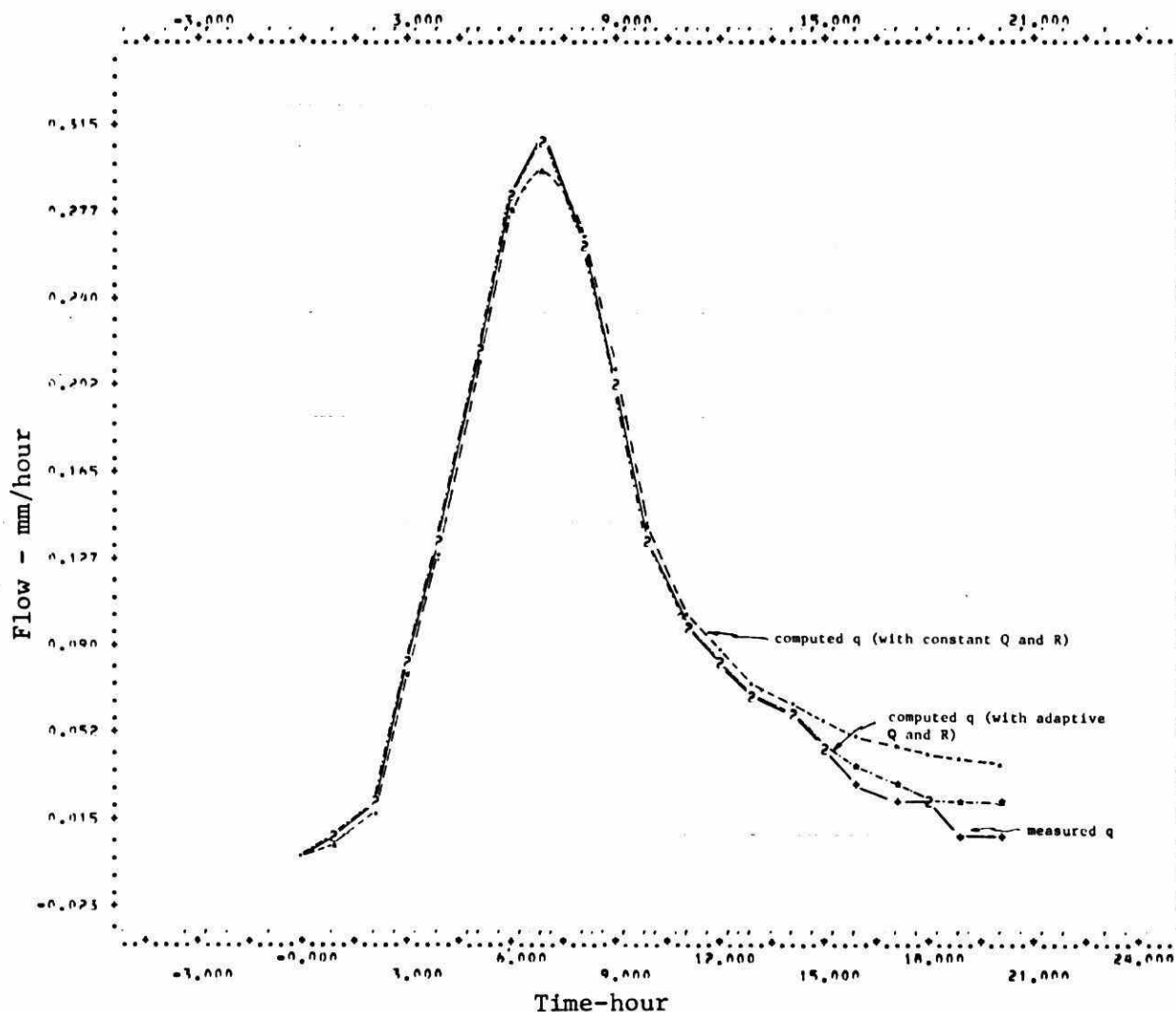


Figure 1.2: Hydrograph Prediction - Sequential Filter Estimation:  
Linear Model (DIUH) (Wilmot Creek Basin - Event 16/8/69).

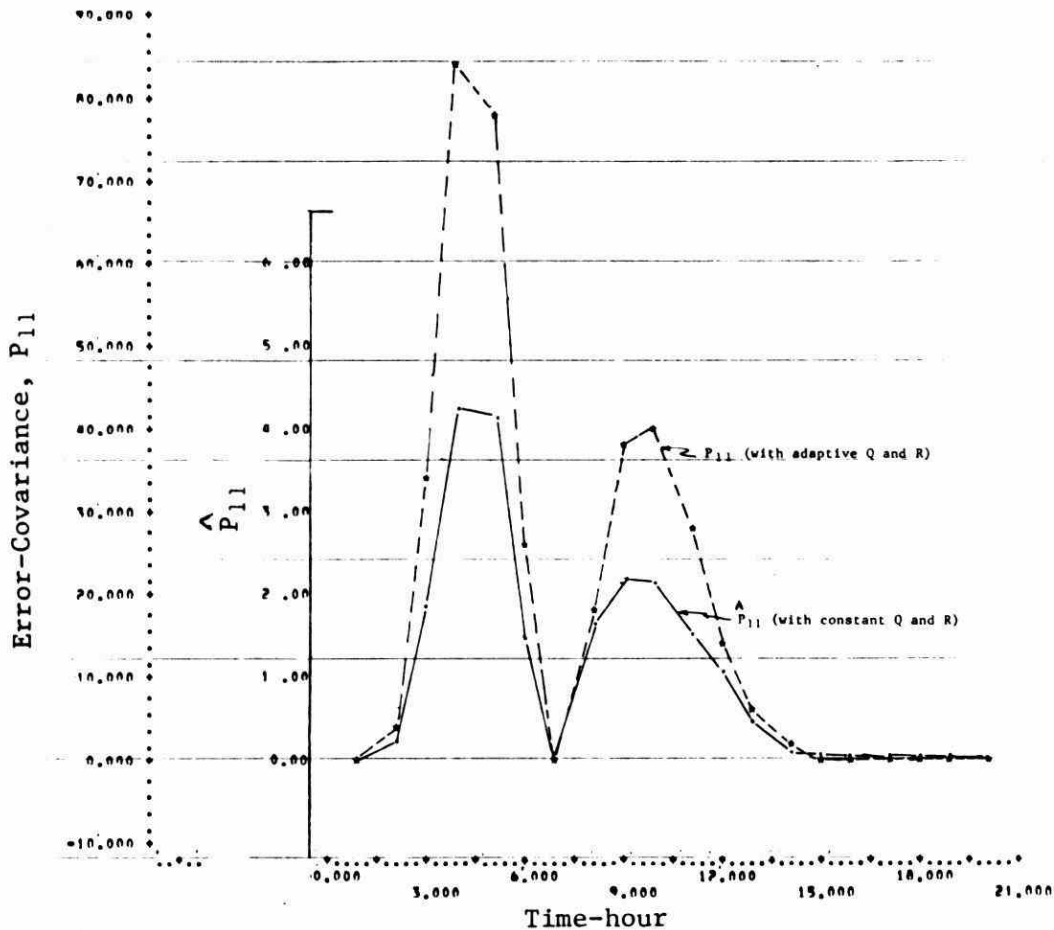


Figure 1.3: Propagation of Error-Covariance,  $P_{11}$ , for Sequential Filter Estimation of Flow,  $q$ ; Linear Model (DIUH) (Wilmot Creek Basin - Event 16/8/69).

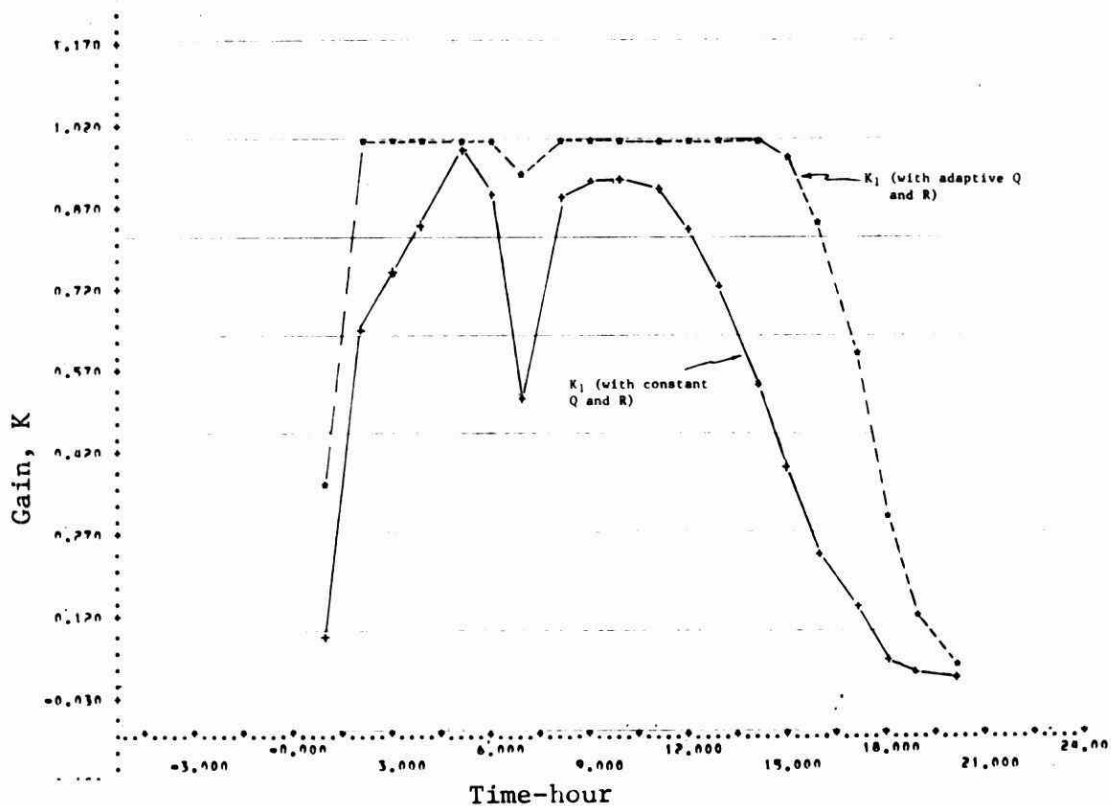


Figure 1.4: Propagation of Kalman Gain,  $K_1$ , for Sequential Filter Estimation of Flow,  $q$ ; Linear Model (DIUH) (Wilmot Creek Basin - Event 16/8/69).

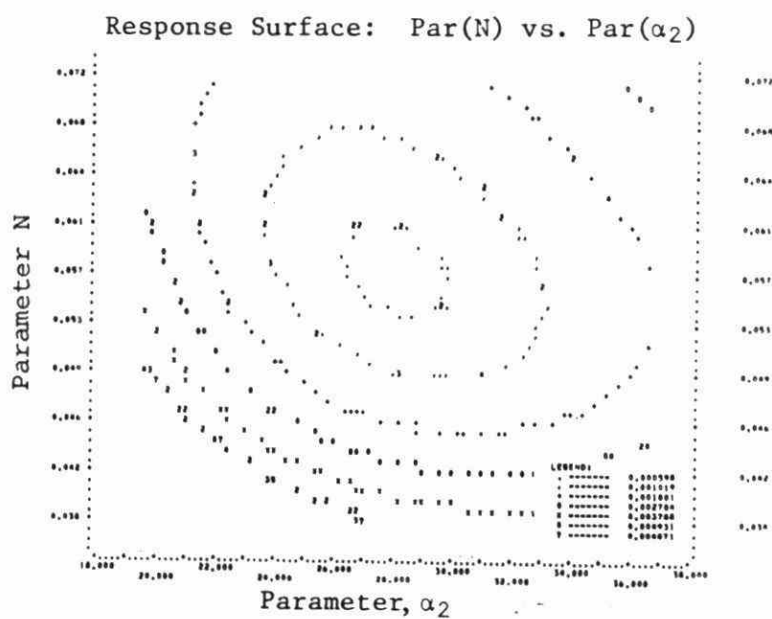
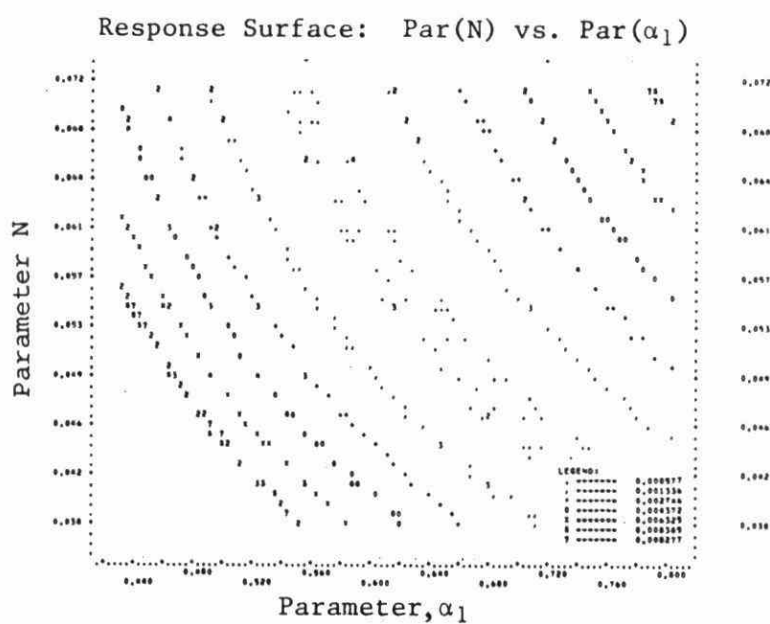
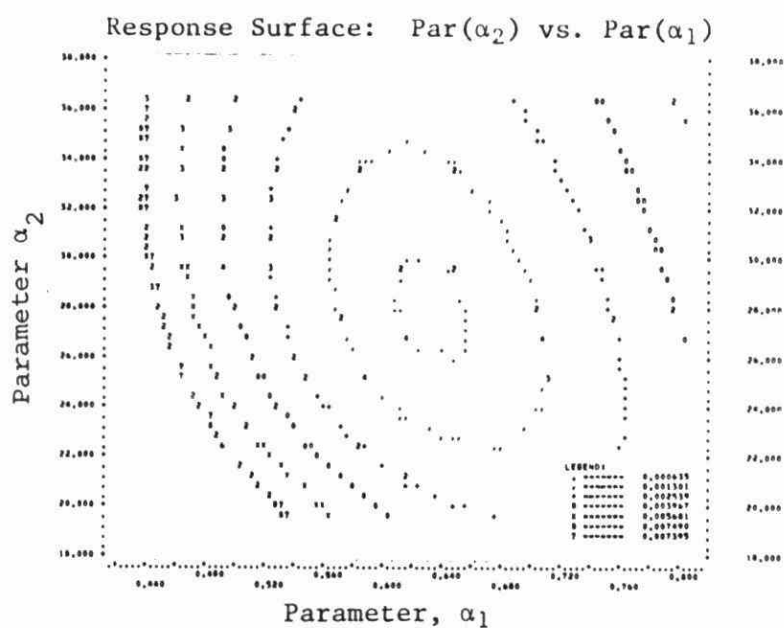


Figure 2.1: Parameter Response Surfaces: Nonlinear Model (Wilmot Creek Basin - Event 20/9/71).

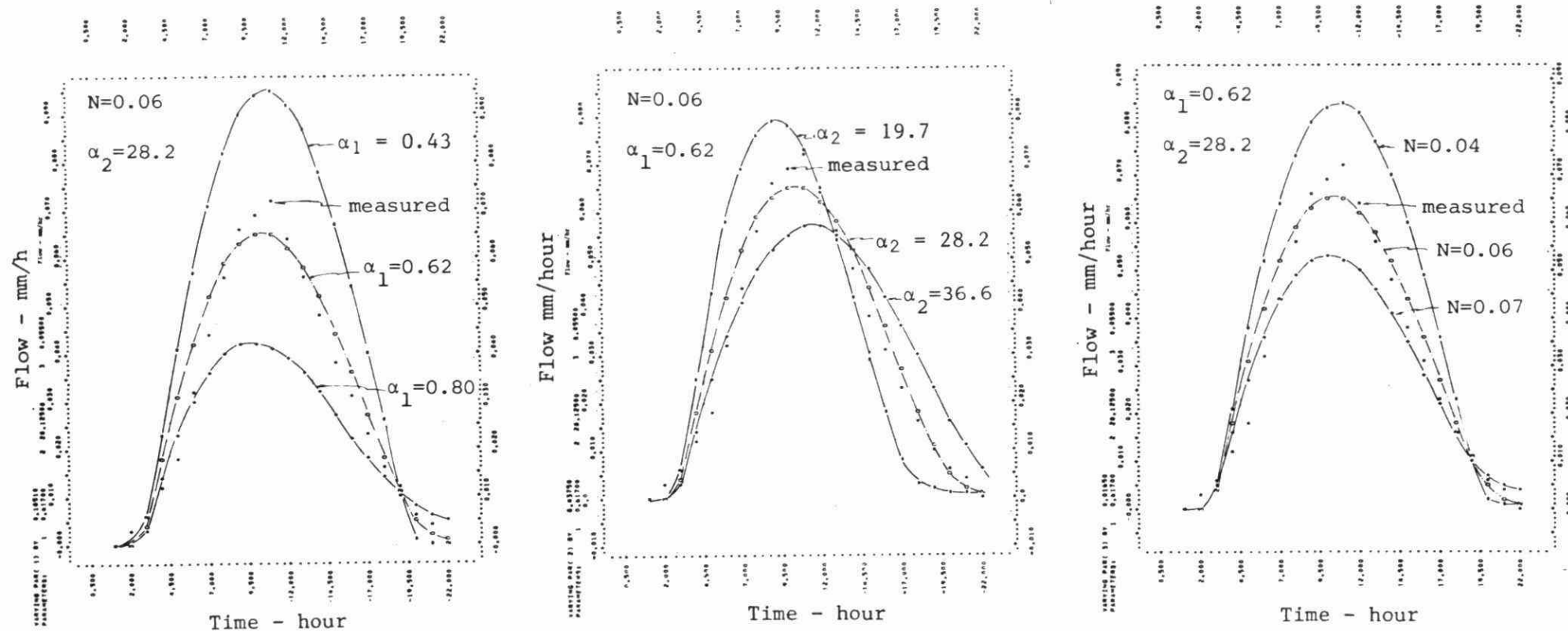


Figure 2.2: Hydrograph Sensitivity to Varying Parameters: Nonlinear Model (Wilmot Creek Basin - Event 20/9/71).

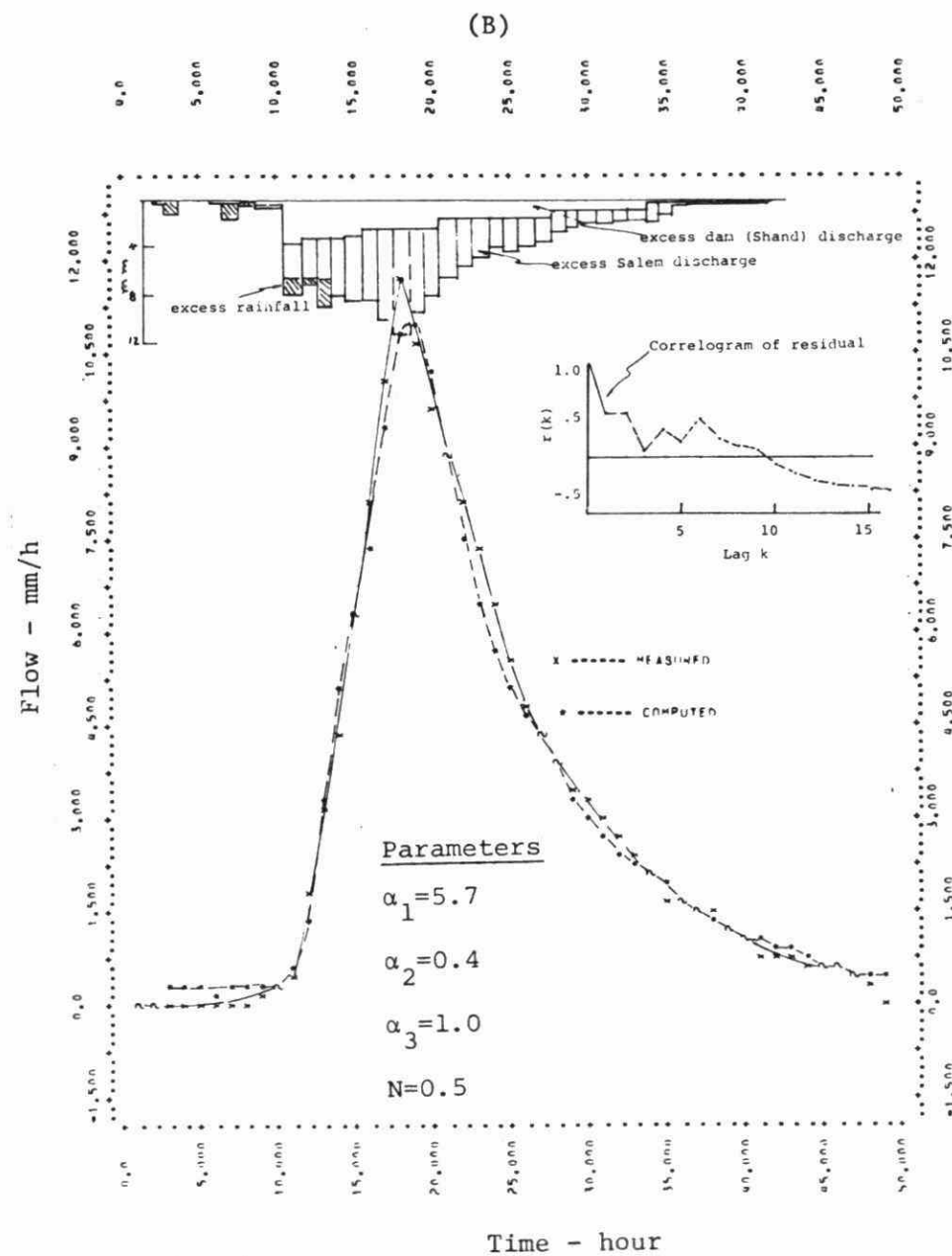
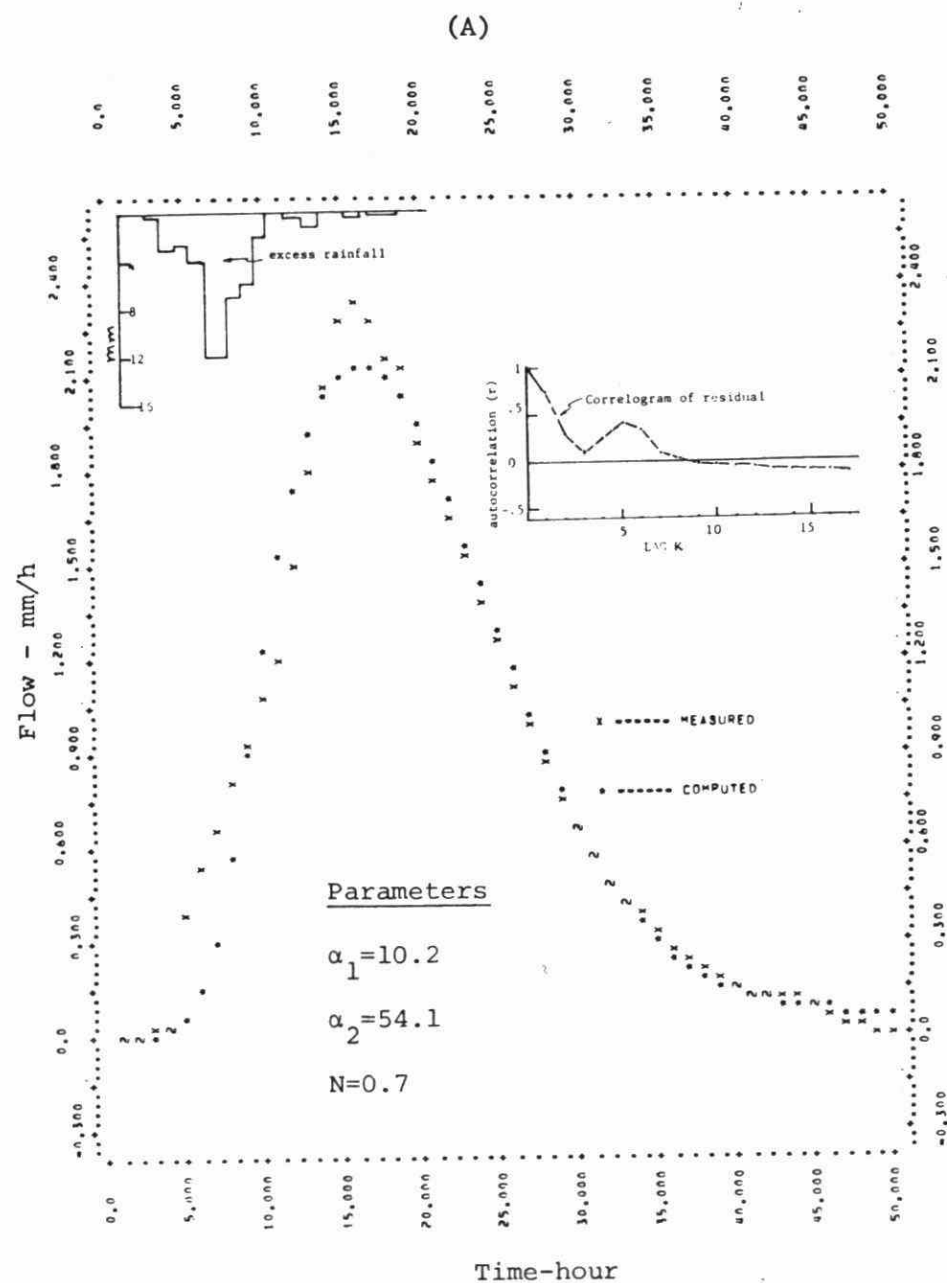
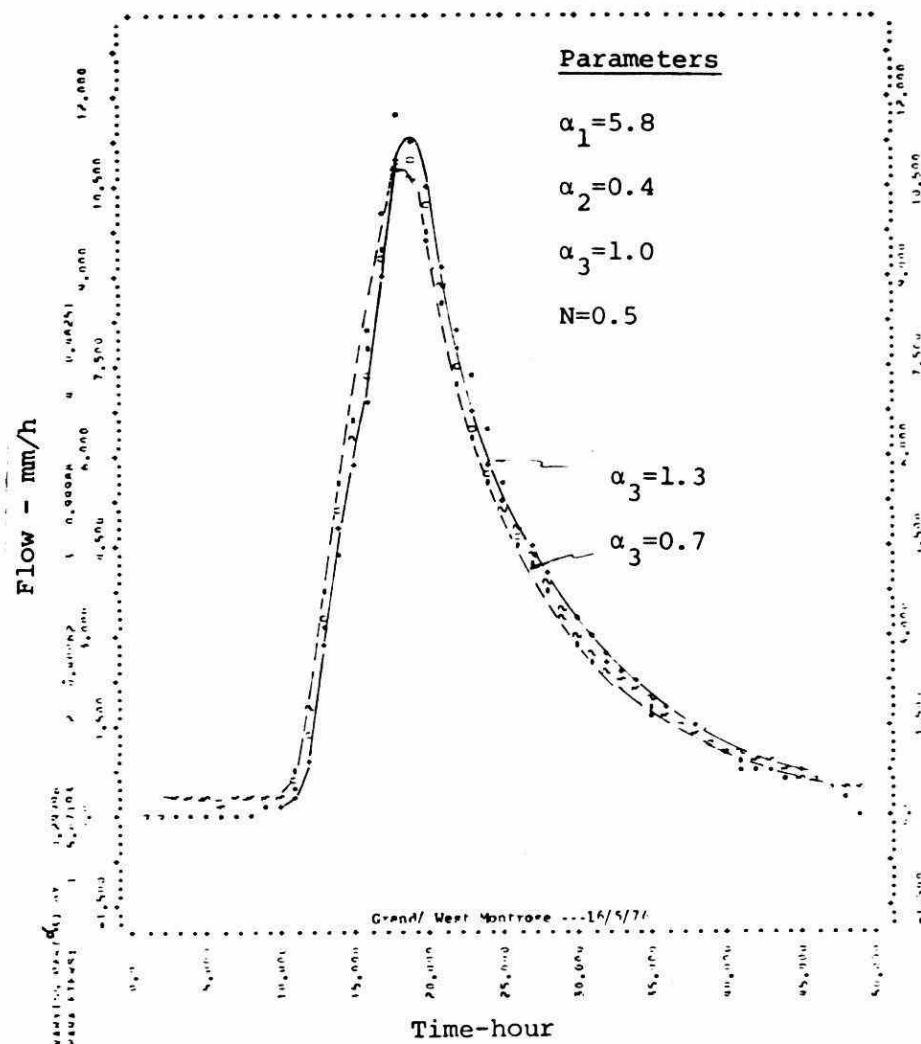


Figure 2.3: Hydrograph Predictions: Nonlinear Model Application (Grand River Basin - Events: (A) - Nithburg 23/8/75, (B) West Montrose 16/5/74).

(A)



(B)

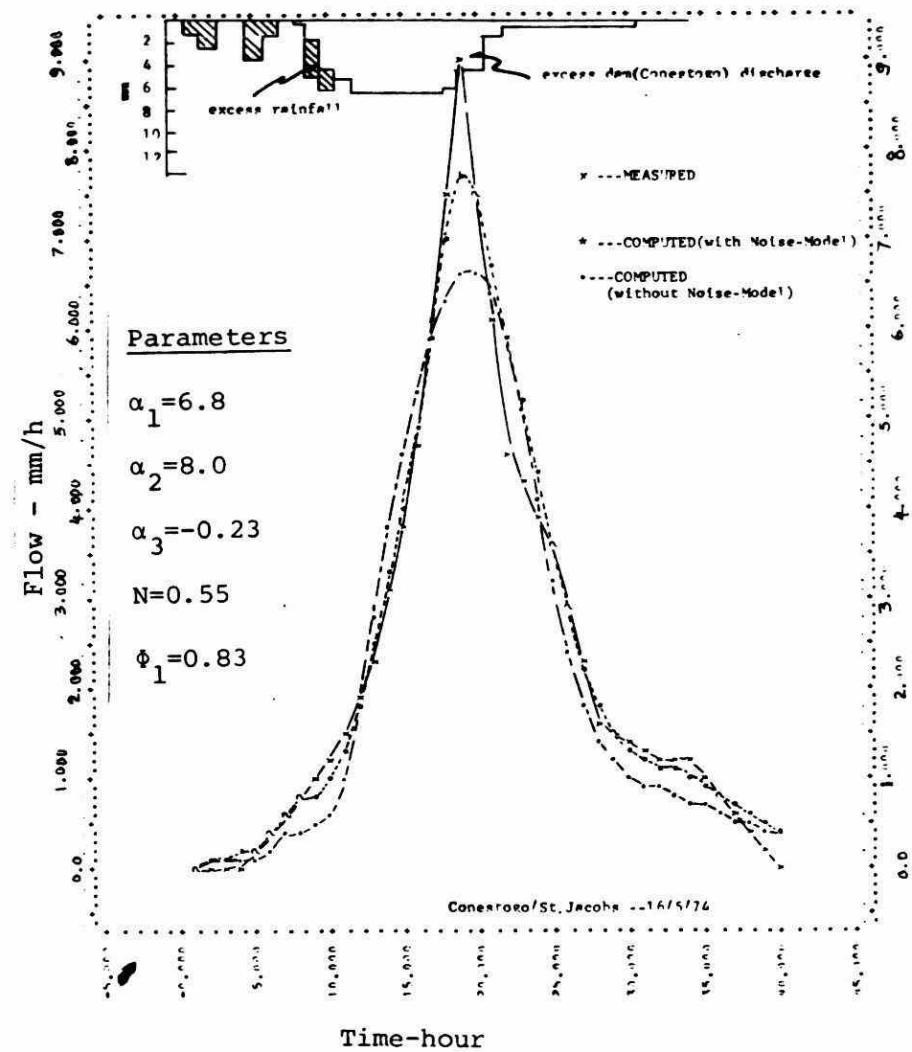


Figure 2.4: Hydrograph Prediction: Nonlinear Model Application - (A) Sensitivity to Varying Parameter  $\alpha_3$  - (B) Effect of Input Control Noise-Model.